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DEPARTMENT OF CIVIL ENGINEERING
AND ENGINEERING MECHANICS

TWO SOLUTIONS FOR THE NONLINEAR ELASTIC THICK WALLED CYLINDER UNDER PRESSURE

by

W. R. Spillers

Office of Naval Research
Project NR 064-446
Contract Nonr 266(78)
Technical Report No. 11
CU-11-61-ONR 266(78) CE

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ABSTRACT

Two methods of solution are applied to the problem of the response of a thick walled cylinder with nonhomogeneous strain-dependent and thus non-linear elastic properties: one method is an application of perturbation theory; the other, an iterative numerical scheme using finite difference approximations. The effects of nonlinearity and the region of validity of the solutions are discussed.

1. Introduction

Recently there has been a great deal of interest in the mechanical response of elastic bodies whose material properties are not homogeneous, but strain-dependent. This interest has been an outgrowth of both the development of new materials and the straining of conventional ones to higher limits. It is the purpose of this report to investigate the effects of small non-linearity on the states of shear and strain in a thick walled cylinder of infinite length (plane strain).

The response of an elastic material may be described by its shear modulus, G, and its bulk modulus K. For this paper it is assumed that the bulk modulus is a linear function of the dilatation, $\varepsilon_{\kappa\kappa}$,

$$K = K_o(1 - \alpha c \varepsilon_{\kappa\kappa}) \tag{1}$$

and that the shear modulus is linearly related to the second deviatoric strain invariant, I'_2 [1],

$$G = G_o(1 - cI'_2) \tag{2}$$

where

$$I'_2 = 1/2 \; e_{ij} e_{ij}$$

in which e_{ij} is the deviatorie strain tensor,

$$e_{ij} = \varepsilon_{ij} - \delta_{ij} \frac{\varepsilon_{\kappa\kappa}}{3}$$

The infinitesimal strain tensor ε_{ij} may be expressed in terms of the displacements, u_i ,

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i})$$

and the summation convention for repeated indicies is employed.

Many derivations of general stress-strain relationships are available in the literature [2,3]. The discussion here will be limited to the observation that the relationships selected, Eqs. (1) and (2), describe a material which is softening under increasing shear strain (I'_2 is always positive and C and α are assumed to be positive constants) but may either soften or harden with regard to volume strain depending upon the sign of the dilatation ε_{KK}. If such a material is subjected to an uniaxial state of stress (as in a tension test) various stress-strain relations may be obtained. Fig. 1 shows such a group of different stress-strain diagrams computed with the use of Eqs. (1) and (2).

It should be noted that while it is the purpose of this report to discuss "small" nonlinearities, which is in fact implied by the form of Eqs. (1) and (2), it is possible to produce "large" nonlinearities by proper selection of the parameters involved.

The analysis of the response of a nonlinear thick walled cylinder (plane strain) enclosed in a linear elastic thin case (see Fig. 5) subjected to an internal pressure p_o can be based on the equilibrium equation expressed in terms of the displacement components. The relations between the principal non-zero components of stress and strain are written in the form

$$
\begin{aligned}
\sigma_r &= (K + \tfrac{4}{3}G)\varepsilon_r + (K - \tfrac{2}{3}G)\varepsilon_\theta \\
\sigma_\theta &= (K - \tfrac{2}{3}G)\varepsilon_r + (K + \tfrac{4}{3}G)\varepsilon_\theta \\
\sigma_z &= (K - \tfrac{2}{3}G)(\varepsilon_r + \varepsilon_\theta)
\end{aligned}
\qquad (3)
$$

The strain displacement relations are

$$\varepsilon_r = u_{,r} \; ; \qquad \varepsilon_\theta = \frac{u}{r} \tag{4}$$

where u denotes the radial displacement component. I'_2 and ε_{KK} in terms of the displacement

$$I'_2 = \frac{1}{3}\left(u_{,r}^2 - u_{,r}\frac{u}{r} + \frac{u^2}{r^2}\right)$$
$$\varepsilon_{KK} = u_{,r} + \frac{u}{r} \tag{5}$$

The equilibrium equation

$$r\sigma_{r,r} + \sigma_r \sigma_\theta = 0 \tag{6}$$

expressed in terms of the displacement, u, becomes

$$(K + \tfrac{4}{3}G)(ru_{,rr} + u_{,r} - \tfrac{u}{r}) + K_{,r}(ru_{,r} + u) + G_{,r}(\tfrac{4}{3}ru_{,r} - \tfrac{2}{3}u) = 0 \tag{7}$$

for the relevant boundary conditions. Methods of solution of this equation will now be disccused.

2. Perturbation Techniques

Perturbation techniques [4] have been shown to be useful in the solution of nonlinear differential equations and other problems (5). Their application involves expansion of the displacement in terms of a parameter c which governs the amount of nonlinearity.

$$u = \sum_{n=0}^{\infty} c^n u_n \tag{8}$$

Introducing Eq. (8) into Eq. (7) and equating the coefficients of each power of c to zero the differential equation for each term u_n is

determined. For u_o and u_1 it is found that

$$(K_o + \tfrac{4}{3}G_o)(ru_{o,rr} + u_{o,r} - \tfrac{u_o}{r}) = 0 \tag{9}$$

or

$$u_o = \frac{c_1}{r} + c_2 r \tag{10}$$

and

$$\begin{aligned}(ru_{1,rr} + u_{1,r} - \tfrac{u_1}{r})(K_o + \tfrac{4}{3}G_o) + \cdot \\ \cdot + \tfrac{1}{3}G_o\left[u_{o,rr}\left(\tfrac{u_o}{r} - 2u_{o,r}\right) + \left(\tfrac{u_{o,r}}{r} - \tfrac{u_o}{r^2}\right)\left(u_{o,r} - 2\tfrac{u_o}{r}\right)\right]\left[\tfrac{4}{3}ru_{o,r} - \tfrac{2}{3}u_o\right] = 0\end{aligned} \tag{11}$$

or

$$u_1 = -G_o c_1^2 \frac{\left(-\frac{c_1}{r^5} + \frac{c_2}{r^3}\right)}{(3K_o + 4G_o)} - k_1\frac{r}{2} - \frac{k_2}{r} \tag{12}$$

where c, c_2, k, and k_2 are constants to be determined from the boundary conditions.

Equation (8) together with Eqs. (5) provide expansions for the stresses in terms of the parameter c,

$$\sigma_i = \sum_{n=0}^{\infty} c^n \sigma_i^{(n)} \qquad (i = r, \theta, z)$$

or, more explicitly,

$$\begin{aligned}
\sigma_r &= \sigma_r^{(0)} + c\sigma_r^{(1)} + c^2\sigma_r^{(2)} + \cdots = K_o\epsilon_{KK}^{(0)} + \tfrac{2}{3}G_o(2\epsilon_r^{(0)} - \epsilon_\theta^{(0)}) + \cdot \\
&\cdot + c\left\{K_o[\epsilon_{KK}^{(1)} - \alpha(\epsilon_{KK}^{(0)})^2] + \tfrac{2}{3}G_o[2\epsilon_r^{(1)} - \epsilon_\theta^{(1)} - I_2'^{(0)}(2\epsilon_r^{(0)} - \epsilon_\theta^{(0)})]\right\} + c^2\{\cdot\} + \cdots \\
\sigma_\theta &= \sigma_\theta^{(0)} + c\sigma_\theta^{(1)} + c^2\sigma_\theta^{(2)} + \cdots = K_o\epsilon_{KK}^{(0)} + \tfrac{2}{3}G_o(-\epsilon_r^{(0)} + 2\epsilon_\theta^{(0)}) + \cdot \\
&\cdot + c\left\{K_o[\epsilon_{KK}^{(1)} - \alpha(\epsilon_{KK}^{(0)})^2] + \tfrac{2}{3}G_o[-\epsilon_r^{(1)} + 2\epsilon_\theta^{(1)} - I_2'^{(0)}(-\epsilon_r^{(0)} + 2\epsilon_\theta^{(0)})]\right\} + c^2\{\cdot\} + \cdots \\
\sigma_z &= \sigma_z^{(0)} + c\sigma_z^{(1)} + c^2\sigma_z^{(2)} + \cdots = (K_o - \tfrac{2}{3}G_o)\epsilon_{KK}^{(0)} + \cdot \\
&\cdot + c\left\{(K_o - \tfrac{2}{3}G_o)\epsilon_{KK}^{(1)} + (-\alpha K_o\epsilon_{KK}^{(0)} + \tfrac{2}{3}G_o I_2'^{(0)})\epsilon_{KK}^{(0)}\right\} + c^2\{\cdot\} + \cdots
\end{aligned}$$

Since the boundary conditions are independent of c, the condition that

$$\sigma_r = -p_o \quad at \quad r = a$$

leads to

$$\left.\begin{aligned}\sigma_r^{(o)} &= -p_o \\ \sigma_r^{(n)} &= 0 \quad for \quad n > 0\end{aligned}\right\} \quad at \quad r = a$$

while the condition that

$$\sigma_r = -\frac{uEt}{b^2(1-\nu^2)} \quad at \quad r = b$$

leads to

$$\sigma_r^{(n)} = -\frac{u_n Et}{b^2(1-\nu^2)} \quad at \quad r = b$$

where E and ν are Young's modulus and Poisson's ratio for the thin outer case.

A number of numerical examples were calculated keeping only the first two terms in each series expansion. Figures 2, 3, 4, and 5 show the results of one such calculation in which $\frac{a}{b} = 0.5$, $\frac{t}{b} = \frac{1}{40}$, $\frac{E}{G_o} = 100$, $\frac{K_o}{G_o} = 1$, $\alpha = 1$ and $\frac{p_o}{G_o}$ and c vary as indicated. It will be shown that this example corresponds to rather large nonlinearity and in this sense is misleading, (see section 4).

3. Iterative Numerical Solutions

The advantage of the previous method of solution is that it provides an analytic solution with relatively little numerical work. One of the

disadvantages is that when only two terms are retained, the results are only valid for "small" nonlinearity.

In this section an iterative procedure is presented using finite difference methods. This procedure is useful as a check on the range of validity of the previous method and also useful in itself as a solution. It is briefly outlined below.

1. The solution for the displacement is approximated, using the solution for the linear elastic thick walled cylinder without an outer shell.

2. This approximation is used to compute the terms in Eq. (7) which involve the elastic constants, thus linearizing the equation.

3. Eq. (7) is then solved as a linear, ordinary differential equation with the aid of finite difference techniques. This solution is now used in place of step 1, and steps 2 and 3 are subsequently repeated.

4. The procedure is stopped when the results have converged sufficiently well.

Appendix 1 presents a Fortran [6] program for this procedure which was used on the IBM 1620 digital computer at the Engineering Computing Center of Columbia University. First the input data, $\frac{t}{b}$, $\frac{K_o}{G_o}$, $-\alpha c$, c, $\frac{p_o}{G_o}$, number of divisions used in the finite difference solution (multiples of 10), and $\frac{tE}{b(1-\nu^2)G_o}$, is read and printed; next the radius $\frac{r}{b}$ and the first approximation for the displacement $\frac{u}{b}$ at that point are printed for a number of points; this is followed by more radius-displacement pairs from each iteration if sense switch 1 is on (if sense switch 1 is off the iterations are not printed) followed by

the number of the iteration in any case; finally sense switch 2 on or off determines whether or not the iteration is stopped. If the iteration is stopped, the final values of $\frac{r}{b}$, $\frac{u}{b}$, $\frac{\sigma_r}{p_o}$, $\frac{\sigma_\theta}{p_o}$, and $\frac{\sigma_z}{p_o}$ are printed for a number of points.

The result of some numerical computations is shown on Fig. 2-5 together with the results of the perturbation solution. For the example shown forty divisions were used in the finite difference solution. The computer running time for each iteration was about two minutes. This increased to about three minutes when fity divisions were used.

Convergence of this iterative procedure is not always insured (a common problem in nonlinear behavior) but it was observed in all cases that by decreasing the nonlinearity, i.e. decreasing c, convergence could be obtained.

4. Discussion of Results

It should be noted that the examples in Figs. 2-5 are not in the region of "small" nonlinearity. For example, the smallest nonlinearity shown has an average K of about seventy percent of K_o. While this amount of nonlinearity is required to produce significant changes in the quantities calculated, it is actually beyond the range of validity of the perturbation method. This accounts for the lack of agreement between the perturbation solution and the iterative solution. On the basis of the numerical analysis performed it is apparent that "small" nonlinearities, which casue less than 10% change in the elastic constants do not

significantly effect the stress field.

The effect of the nonlinearity is to decrease the tangential stress at the inside surface, an effect observed in an elastic plastic thick walled cylinder and also in some nonhomogeneous problems (5). Other effects include a general softening of the material and a corresponding redistribution of stress between the thin outer shell and the cylinder.

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6. IBM 1620 Fortran: Preliminary Specifications (This is an IBM Bulletin).

Appendix 1

```
DIMENSION R(53),U(53),D(53),E(53),F(53)
12  READ,T,AKOG,A,B,POG,DIV,SHELL
PRINT,T,AKOG,A,B,POG,DIV ,SHELL
AHOB=(1.-T)/DIV
C8=0.
M=DIV+3.
13  DO 15 I=1,M
Z8=I-2
14  R(I)=T+(Z8  )*(1.-T)/DIV
U(I)=POG*.5*(1./R(I)+R(I)/(.33333333+AKOG))/(1./(T*T)-1.)
Y8=Z8-C8*.1*DIV
IF (Y8)15,96,96
96  C8=C8+1.
  PRINT,R(I),U(I)
15  CONTINUE
I=1
16  K=2
17  L=-1
18  DU=.5 *(U(K+1)-U(K-1))/AHOB
19  A12=(-DU*DU+DU*U(K)/R(K)-U(K)*U(K)/(R(K)*R(K)))
20  G=1.+B*A12*.33333333
AK=AKOG*(1.+A*(DU+U(K)/R(K)))
IF (L) 21,21,55
21  DDU=     (U(K+1)+U(K-1)-2.*U(K))/( AHOB*AHOB    )
22  G1=DDU*(U(K)/R(K)-2.*DU)
23  G2=(DU-U(K)/R(K))*(DU-2.*U(K)/R(K))/R(K)
24  DG=B*.33333333*(G1+G2)
25  DAK=AKOG*A*((DU-U(K)/R(K))/R(K )+DDU)
26  IF (L) 27,33,62
27  D(1)=(AK+1.333333333*G)*.5/AHOB
28  F(1)=(AK-.66666666*G)/R(2)
29  E(1)=-D(1)
N=M-1
30  DO 37 K=2,N
31  L=0
32  GO TO 18
33  F1=AK+1.33333333*G
34  F2=.5*R(K)*(DAK+1.3333333333*DG)
35  F(K)=    (F1*(R(K)/AHOB      +.5)+F2)/AHOB
36  E(K)=F1*(-R(K)*2./(AHOB*AHOB)       -1./R(K))-.66666666*DG+DAK
37  D(K)=             (F1*(R(K)/AHOB       -.5)-F2)/AHOB
38  E(M )=(AK+1.33333333*G)*.5/AHOB
39  D(M )=(AK-.66666666*G)/R(N )+SHELL
40  F(M )=-E(M )
41  E(N)=E(N)-D(M)*F(N)/E(M)
D(N)=D(N)-F(M)*F(N)/E(M)
42  DO43 K=3,N
J=M-K+1
43  E(J)=E(J)-D(J+1)*F(J)/E(J+1)
F(1)=F(1)-D(3)*D(1)/E(3)
E(1)=E(1)-D(2)*F(1)/E(2)
44  U(1)=-POG/E(1)
45  DO 46 K=2,N
46  U(K)=-D(K)*U(K-1)/E(K)
47  U(M)=-(F(M)*U(N-1)+D(M)*U(N))/E(M)
IF (SENSE SWITCH 1) 48,51
```

```
48 M2=.1*DIV
    DO 49 K=2,N,M2
49  PRINT,R(K),U(K)
51  PRINT,I
I=I+1
53  IF (SENSE SWITCH 2)52,16
52  DO 60 K=2,N,M2
53  L=+1.
54  GO TO 18
55  X1=AK+1.33333333*G
56  X2=AK-.66666666*G
57 X=(X1*DU+X2*U(K)/R(K))/POG
58 Y=(X2*DU+X1*U(K)/R(K))/POG
59  Z=X2*(DU+U(K)/R(K))/POG
60  PRINT,R(K),U(K),X,Y,Z
61  GO TO 12
62  END
```

UNIAXIAL STRESS IN A NONLINEAR ROD

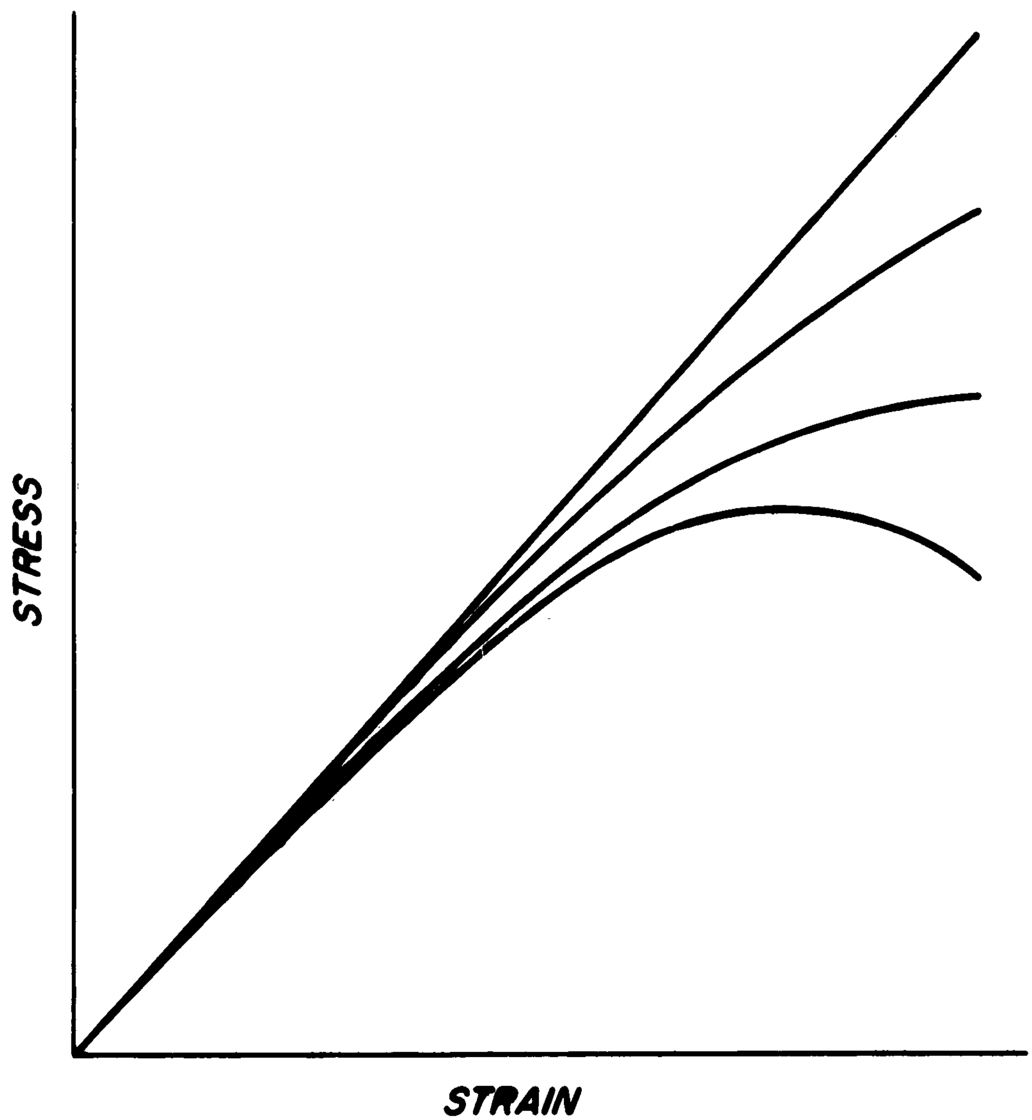
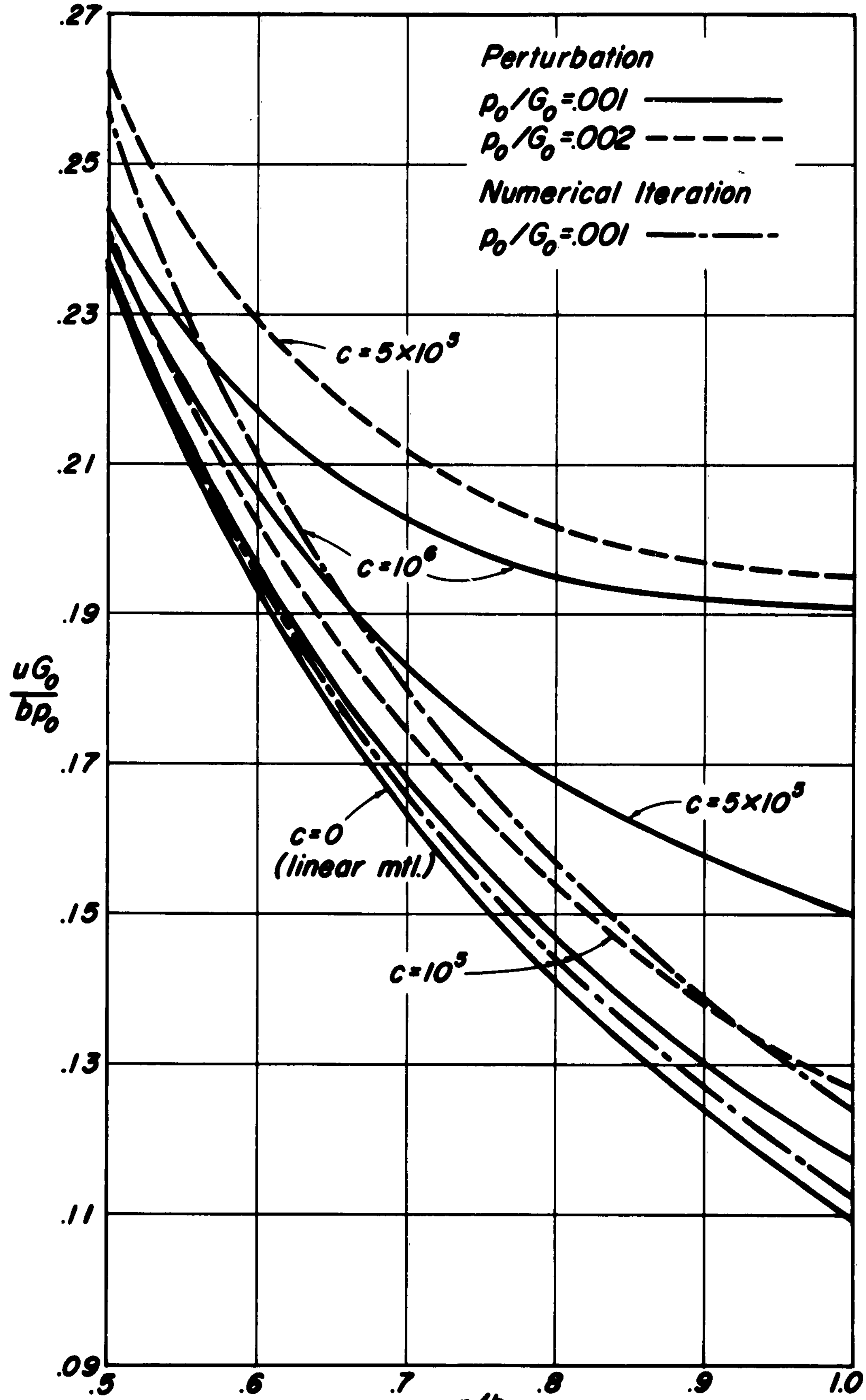
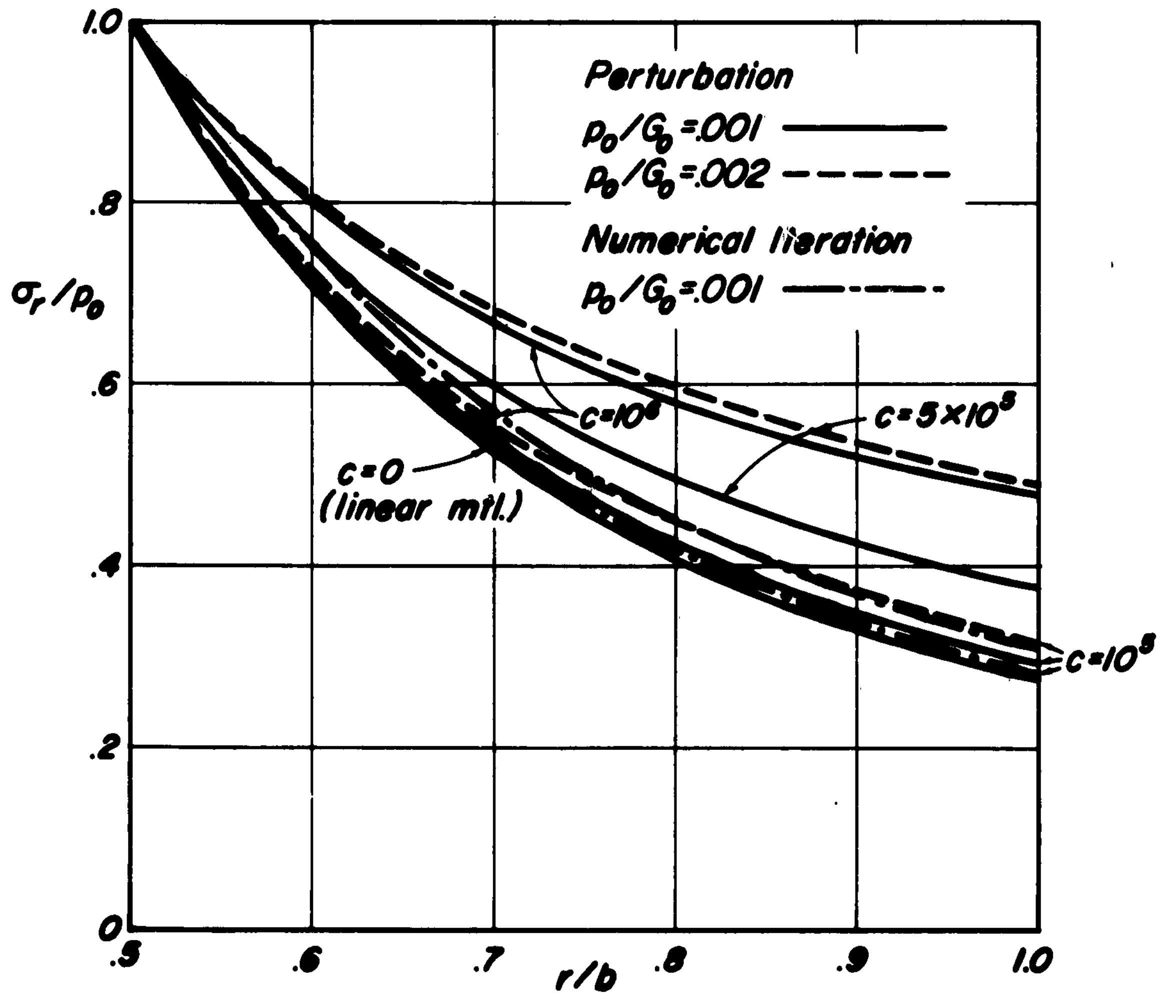


FIG. 1



Perturbation
p_0/G_0=.001
p_0/G_0=.002
Numerical Iteration
p_0/G_0=.001
c=5×10⁵
c=10⁶
c=5×10⁵
c=0
(linear mtl.)
c=10⁵
.27
.25
.23
.21
.19
.17
.15
.13
.11
.09
$\frac{uG_0}{bp_0}$
.5
.6
.7
.8
.9
1.0
r/b

FIG.2



1.0
.8
.6
.4
.2
0
σ_r / p_0
Perturbation
p_0/G_0=.001
p_0/G_0=.002
Numerical iteration
p_0/G_0=.001
c=10^6
c=5×10^5
c=0
(linear mtl.)
c=10^5
.5
.6
.7
.8
.9
1.0
r/b

FIG. 3

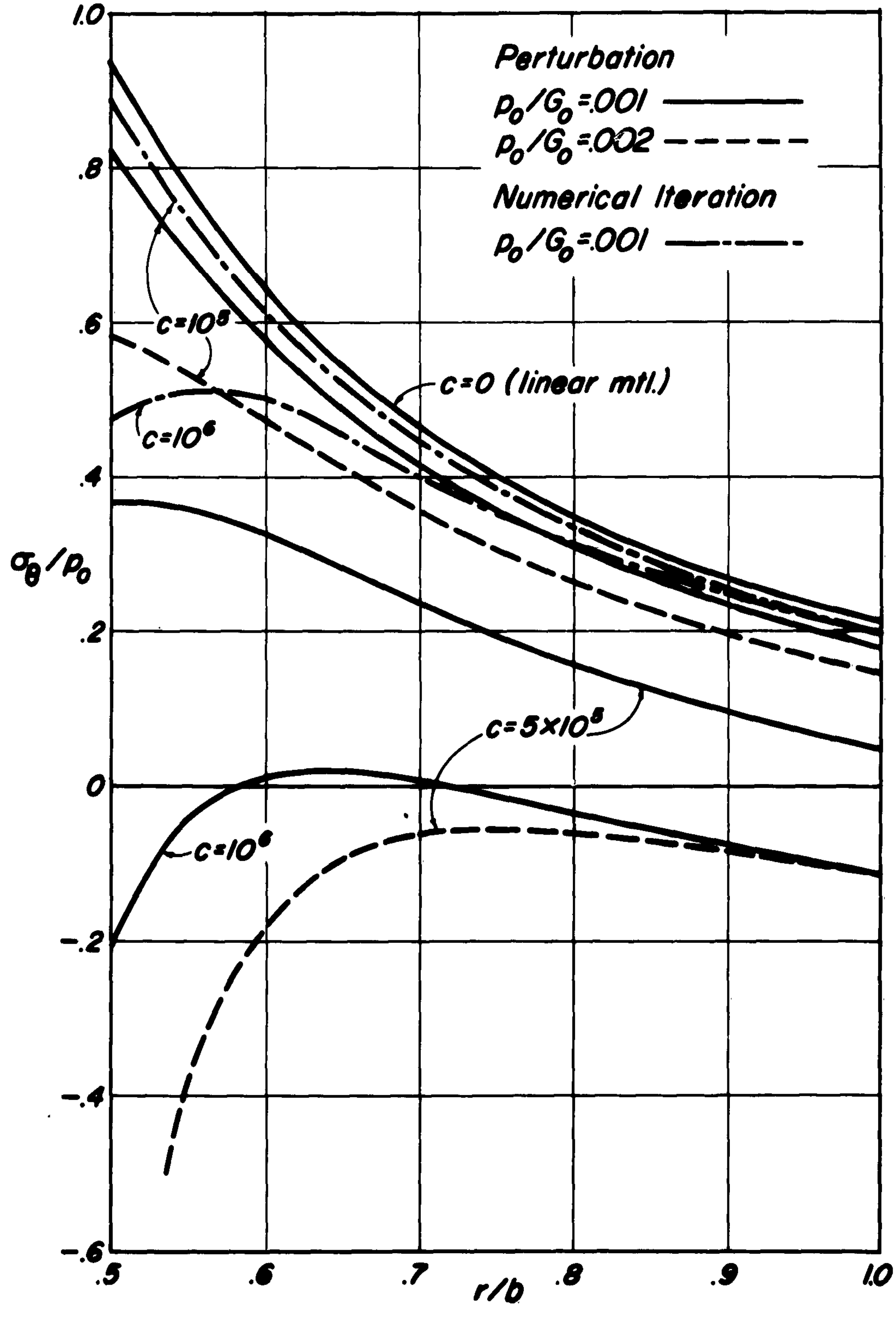
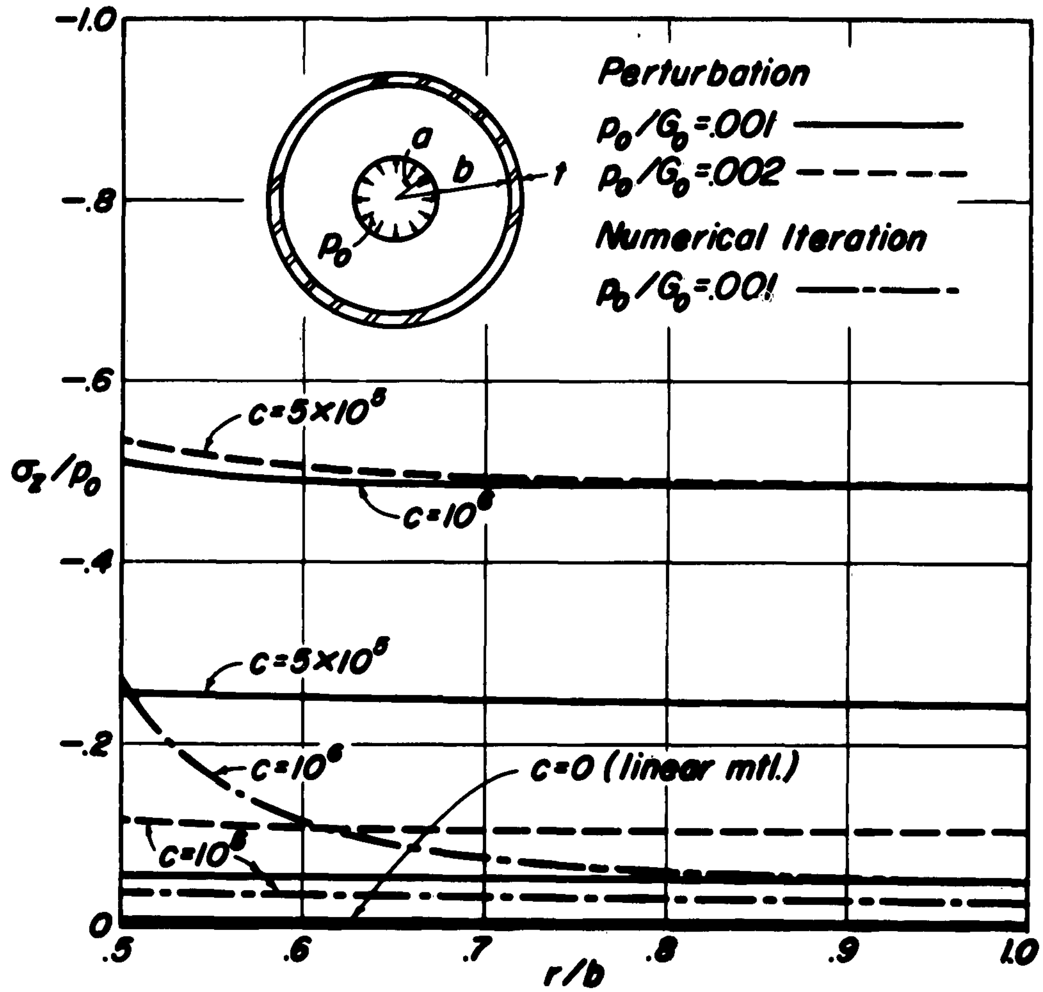

Perturbation
p0/G0=.001
p0/G0=.002
Numerical Iteration
p0/G0=.001
c=10^5
c=10^6
c=0 (linear mtl.)
c=5×10^5
c=10^6
σθ/p0
r/b
1.0
.8
.6
.4
.2
0
−.2
−.4
−.6
.5
.6
.7
.8
.9
1.0

FIG. 4



Perturbation
$p_0/G_0 = .001$
$p_0/G_0 = .002$
Numerical Iteration
$p_0/G_0 = .001$
a
b
1
p_0
$c = 5 \times 10^5$
$c = 10^6$
$c = 5 \times 10^5$
$c = 10^6$
$c = 0$ (linear mtl.)
$c = 10^5$
σ_z/p_0
r/b
-1.0
-.8
-.6
-.4
-.2
0
.5
.6
.7
.8
.9
1.0

FIG. 5

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Picatinny Arsenal
Dover, New Jersey

U.S. Army Research Office
2127 Myrtle Drive
Duke Station
Durham, North Carolina
Attn: Div. of Engineering Sciences

Chief of Naval Operations
Department of the Navy
Washington 25, D.C.
Attn: Op 07T
Op 03EG

Quality Evaluation Laboratory
Naval Ammunition Depot
Concord, California
Attn: D.R. Smathers

U.S. Naval Ordnance Laboratory
Non Metallic Materials Division
Silver Spring, Maryland
Attn: H.A. Perry, Jr.

U.S. Naval Ordnance Test Station
China Lake, California
Attn: J.T. Bartling

U.S. Naval Propellant Plant
Indian Head, Maryland
Attn: J. Browning
L. Papier (L)

Office of Naval Research
Branch Office
495 Summer Street
Boston 10, Massachusetts
Attn: Dr. J.H. Faull, Jr.

National Aeronautics & Space Adm.
1570 H Street, N.W.
Washington 25, D. C.
Attn; Chief, Div. of Research
Information

<u>Contractors</u>

Atlantic Research, Inc.
Shirley Highway and Edsall Road
Alexandria, Virginia
Attn: M.G. DeFries

Battelle Memorial Institute
505 King Avenue
Columbus 1, Ohio
Attn: J.Harry Jackson

Brown University
Division of Applied Mathematics
Providence 12, Rhode Island
Attn: Prof. E.H. Lee
Prof. R.S. Rivlin

University of California
College of Engineering
Berkeley 4, California
Attn: Prof. Paul M.Naghdi

Catholic University of America
Department of Civil Engineering
620 Michigan Avenue, N.E.
Washington, D.C.
Attn: Prof. A.J. Durelli

Columbia University
Department of Civil Engineering
Amsterdam Avenue & 120th Street
New York 27, New York
Attn: Prof.A.M. Freudenthal

Materials Technology, Inc.
11 Leon Street
Boston 15, Massachusetts
Attn: Dr. R.G. Cheatham

New York University
Depart. of Aeronautical Engineering
University Heights
New York 53, New York
Attn: Prof. H. Becker

University of Pennsylvania
Graduate Division of Engineering Mechanics
Philadelphia 4, Pennsylvania
Attn: Prof. Z. Hashin

Polytechnic Institute of Brooklyn
333 Jay Street
Brooklyn 1, New York
Attn: Prof. F. Romano
Prof. J. Klosner
Prof. F. Ullman

Southwest Research Institute
8500 Culebra Road
San Antonio 6, Texas
Attn: Dr. R.C. DeHart

Central Laboratory T. N. O.
134 Julianalaan
Delft, Holland
Attn: Dr. F. Schwarzl

Aerojet-General Corporation
Solid Rocket Plant
Sacramento, California
Attn: Dr. W.O. Wetmore (2)

Aerojet-General Corporation
P.O. Box 1168
Sacramento, California
Attn: A. Fraser
L.H. Linder

Amcel Propulsion, Inc.
Box 3049
Asheville, North Carolina
Attn: R.N. Lowrey

American Cyanamid Company
1937 West Main Street
Stamford, Connecticut
Attn: Dr. V. Wystrach

University of California
Berkeley, California
Attn: Dr. K.S. Pister

Catholic University of America
Department of Civil Engineering
Washington, D.C.
Attn: J. Baltrukonis

University of Florida
College of Engineering
Gainesville, Florida
Attn: J. Griffith

University of Illinois
Department of Aero Engineering
Urbana, Illinois
Attn: Dr. H.H. Hilton

E.I. duPont de Nemours and Co.
Gibbstown, New Jersey
Attn: R.D. Spangler

Grand Central Rocket Company
P.O. Box 111
Redlands, California
Attn: A.T. Camp

Hercules Powder Company
Allegany Ballistics Laboratory
Cumberland, Maryland
Attn: Dr. R. Steinberger (2)

Jet Propulsion Laboratory
4800 Oak Grove Drive
Pasadena 3, California
Attn: G. Lewis

Lockheed Missile & Space Company
1122 Jagels Road
Sunnyvale, California
Attn: E. Luken (2)

North American Aviation
Rocketdyne Division
6633 Canoga Avenue
Canoga Park, California
Attn: F. Cramer

Rohm & Haas Company
Redstone Arsenal Research Division
Huntsville, Alabama
Attn: H. Shuey

Thiokol Chemical Corporation
Redstone Division
Huntsville, Alabama
Attn: Technical Director
J. Wise

PART IV - Activities and contractors concerned with other aspects of Elastomer Mechanics

Government

Commanding Officer
Office of Naval Research
Branch Office
John Crerar Library Building
86 E. Randolph Street
Chicago 11, Illinois

Commanding Officer
Office of Naval Research
Branch Office
346 Broadway
New York 13, New York

Commanding Officer
Office of Naval Research
Branch Office
1030 E. Green Street
Pasadena, California

Commanding Officer
Office of Naval Research
Branch Office
1000 Geary Street
San Francisco, California

Commanding Officer
Office of Naval Research
Branch Office
Navy 100, Fleet Post Office
Box 39 FPO
New York, New York (5)

Director
Naval Research Laboratory
Washington 25, D.C.
Attn: Tech. Info. Officer (6)
Code 6200
Code 6210

Chief, Bureau of Ships
Department of the Navy
Washington 25, D.C.
Attn: Code 335

Professor R.L. Bisplinghoff
Department of Aeronautical Engineering
Massachusetts Institute of Technology
Cambridge 39, Massachusetts

Chief, Bureau of Yards & Docks
Department of the Navy
Washington 25, D.C.
Attn: Code 70

Commanding Officer & Director
David Taylor Model Basin
Washington 7, D.C.
Attn: Code 700

Director
Materials Laboratory
New York Naval Shipyard
Brooklyn 1, New York

Officer-in-Charge
Naval Civil Engineering Research
and Evaluation Laboratory
U.S. Naval Construction Battalion
Center
Port Hueneme, California

Commander
U.S. Naval Proving Ground
Dahlgren, Virginia

Commanding Officer & Director
U.S. Naval Engineering Experiment
Station
Annapolis, Maryland

Superintendent
U.S. Naval Postgraduate School
Monterey, California

National Sciences Foundation
1520 H Street, N.W.
Washington, D.C.
Attn: Engineering Sci. Div.

Professor H.H. Bleich
Department of Civil Engineering
Columbia University
Amsterdam Avenue & 120th Street
New York 27, New York

Professor B.A. Boley
Department of Civil Engineering
Columbia University
Amsterdam Avenue & 120th Street
New York 27, New York

Professor B. Budiansky
Department of Mechanical Engineering
School of Applied Sciences
Harvard University
Cambridge 38, Massachusetts

Professor G. F. Carrier
Pierce Hall
Harvard University
Cambridge 38, Massachusetts

Professor D.C. Drucker
Division of Engineering
Brown University
Providence 12, Rhode Island

Professor J. Ericksen
Mechanical Engineering Department
Johns Hopkins University
Baltimore 18, Maryland

Professor A.C. Eringen
Department of Aeronautical Engineering
Purdue University
Lafayette, Indiana

Mr. Martin Goland, President
Southwest Research Institute
8500 Culebra Road
San Antonio 6, Texas

Professor J.N. Goodier
Department of Mechanical Engineering
Stanford University
Stanford, California

Professor P.G. Hodge
Department of Mechanics
Illinois Institute of Technology
Chicago 16, Illinois

Professor N.J. Hoff, Head
Division of Aeronautical Engineering
Stanford University
Stanford, California

Professor J. Kempner
Dept. of Aeronautical Engineering
and Applied Mechanics
Polytechnic Institute of Brooklyn
333 Jay Street
Brooklyn, New York

Professor R.D. Mindlin
Dept. of Civil Engineering
Columbia University
Amsterdam Avenue & 120th Street
New York 27, New York

Professor William A. Nash
Dept. of Engineering Mechanics
University of Florida
Gainesville, Florida

Professor N.M. Newmark, Head
Dept. of Civil Engineering
University of Illinois
Urbana, Illinois

Professor E. Reiss
Institute of Mathematical Sciences
New York University
25 Waverly Place
New York 3, New York

Professor W. Prager, Chairman
Physical Sciences Council
Brown University
Providence 12, Rhode Island

Professor E. Reissner
Dept. of Mathematics
Massachusetts Institute of Technology
Cambridge 39, Massachusetts

Professor Bernard W. Shaffer
Dept. of Mechanical Engineering
New York University
University Heights
New York 53, New York

Professor Eli Sternberg
Dept. of Mechanics
Brown University
Providence 12, Rhode Island

Professor A.S. Velestos
Dept. of Civil Engineering
University of Illinois
Urbana, Illinois